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THE VOIDAGE FUNCTION FOR FLUID–PARTICLE INTERACTION SYSTEMS

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Abstract—The drag force on a particle in a fluid–multiparticle interaction system may be expressed as the product of the drag force on an unhindered particle, subject to the same volumetric flux of fluid, and a voidage function. It is demonstrated that for a wide variety of both fixed-bed and suspended-particle systems, the voidage function may be expressed as $\varepsilon^{-\beta}$, where the exponent β is dependent on the particle Reynolds number but independent of other system variables.

Key Words: fluid–particle interaction, voidage function, fluidization, sedimentation, fixed beds

1. INTRODUCTION

We are concerned here with the rather general problem of a multiparticle system swept by a fluid: the particles may be maintained in a fixed orientation, either by direct contact amongst themselves or by some other means; or they may be suspended in a fluidized bed or be falling through the fluid in a sedimenting process.

Wallis (1969) has presented the general one-dimensional momentum equation for the particle phase as follows:

$$\rho_p\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}\right)=b+f-\frac{\partial p}{\partial z}; \qquad [1]$$

where, for all the forces on the right-hand side, b represents all the body forces, $-\partial p/\partial z$ is the pressure gradient force, which, in general, should include contributions from both the fluid and particle pressures, and f accounts for all the surface forces acting on the solid phase that are not included in the pressure gradient term. Under uniform, steady-state conditions the left-hand side of [1] vanishes, and for most practical purposes b reduces to the gravitational force; p, in the $\partial p/\partial z$ term, represents the fluid pressure; and f consists of the sum of the hydrodynamic drag and any direct contact force (from, for example, neighbouring particles).

The terms in [1] represents forces per unit volume of the particle phase; multiplying by the volume, V, of a single particle and employing the above considerations yields the steady-state force balance for a single particle:

$$-V\rho_p g+F-V\frac{\mathrm{d}p}{\mathrm{d}z}=0, \qquad [2]$$

where F is the sum of the single particle drag and contact forces,

$$F=F_D+F_C. \qquad [3]$$

In the absence of other particles, the force on a single particle exposed to a relative fluid flux u is $F_D=F_{D0}$, where

$$F_{D0}=C_{D0}\frac{\rho u^2}{2}\frac{\pi d^2}{4}. \qquad [4]$$

For spherical particles many well-established correlations are available for the drag coefficient C_{D0}: that due to Dallavalle,

$$C_{D0} = \left(0.63 + \frac{4.8}{\mathrm{Re}^{0.5}}\right)^2, \qquad [5]$$

provides an adequate representation of the available empirical data over the full practical range of Re.

The purpose of this paper is to examine how F_D deviates from F_{D0}, [4], when the flow field is modified by the presence of other particles. Other interactions, "wall effects" etc., are not considered.

2. THE VOIDAGE FUNCTION

We start with the working hypothesis that the effect of the neighbouring particles on the drag force experienced by a particular particle may be considered solely as a function of the local volumetric particle concentration, or void fraction, ε:

$$F_D = F_{D0}\, g(\varepsilon). \qquad [6]$$

In [6] it is understood that F_D and F_{D0} are to be evaluated at the same value of the fluid flux, u, and hence at the same particle Reynolds number, $\mathrm{Re} = u\rho d/\eta$. Ideally we would like to obtain theoretical expressions for $g(\varepsilon)$; in practice, this approach appears limited to laminar flow through either very dilute suspensions (Batchelor 1972) or regular fixed arrays of spheres (Happel 1958). Experimental data from which $g(\varepsilon)$ can be evaluated numerically are, however, abundant for both fixed and suspended-particle systems over the full practical range of flow regime and particle concentration.

It is necessary to admit openly at this stage that there appears to be no *a priori* reason for believing the voidage function, $g(\varepsilon)$, to be the same for physical systems as varied as, say, densely packed beds and dilute fluidized suspensions: the fact that the evidence presented below points to this convenient generalization is therefore surprising.

3. NUMERICAL EVALUATION OF THE VOIDAGE FUNCTION

3.1. Fluidized and Sedimenting Beds

In this case there are no contact forces, so $F = F_D$. The pressure gradient for an equilibrium fluidized bed is given by

$$\frac{dp}{dz} = -[(1-\varepsilon)\rho_p + \varepsilon\rho]g. \qquad [7]$$

Under these conditions, [2] yields

$$F_D = \frac{\pi d^3}{6}(\rho_p - \rho)g\varepsilon. \qquad [8]$$

Equation [8] enables F_D to be related to F_{Dt}, the drag on a single particle under terminal fluidization conditions, $u = u_t$, $\varepsilon = 1$:

$$F_D = F_{Dt}\,\varepsilon. \qquad [9]$$

In terms of the terminal condition drag coefficient, C_{Dt}, [9] becomes

$$F_D = C_{Dt}\frac{\rho u_t^2}{2}\frac{\pi d^2}{4}\varepsilon. \qquad [10]$$

Expressing also F_{D0} in terms of a drag coefficient, [4], leads to the general expression for the voidage function for fluidized and sedimenting beds:

$$g(\varepsilon) = \frac{F_D}{F_{D0}} = \frac{C_{Dt}}{C_{D0}}\left(\frac{u}{u_t}\right)^{-2}\varepsilon. \qquad [11]$$

It should be emphasized that the two drag coefficients in [11] both relate to single, unhindered particle systems and so may be evaluated using, for example, [5].

To obtain numerical evaluations of $g(\varepsilon)$, data relating the fluidizing flux, u, to the void fraction, ε, are necessary. These can be provided by the empirical Richardson–Zaki equation

$$\frac{u}{u_t} = \varepsilon^n, \qquad [12]$$

where the exponent, n, is a well-documented function of the terminal Reynolds number, Re_t, evaluated at velocity u_t (Richardson & Zaki 1984).

3.1.1. The low Re regime

Under these conditions we have

$$\frac{C_{Dt}}{C_{D0}} = \frac{\dfrac{24}{\mathrm{Re}_t}}{\dfrac{24}{\mathrm{Re}}} = \frac{u}{u_t} \qquad [13]$$

and

$$g(\varepsilon) = \left(\frac{u}{u_t}\right)^{-1} \varepsilon. \qquad [14]$$

Applying [12] and the empirical value for the exponent n for low Reynolds number conditions, $n = 4.65$, yields

$$g(\varepsilon) = \varepsilon^{1-n} = \varepsilon^{-3.65}. \qquad [15]$$

3.2.1. The high Re regime

In this regime the drag coefficient is independent of velocity:

$$\frac{C_{Dt}}{C_{D0}} = 1. \qquad [16]$$

Thus,

$$g(\varepsilon) = \left(\frac{u}{u_t}\right)^{-2} \varepsilon. \qquad [17]$$

n is also constant in this regime and equal to 2.35, thus

$$g(\varepsilon) = \varepsilon^{1-2n} = \varepsilon^{-3.7}. \qquad [18]$$

These results, [15] and [18], are well-known and the fact that the voidage function turns out to be identical, for practical purposes, for extreme conditions has led to the adoption of the same expression for the intermediate flow regime as well (Wen & Yu 1966; Wallis 1969; Richardson & Jeronimo 1976; Foscolo *et al.* 1983).

Kahn & Richardson (1990) have pointed out that such a generalization is inconsistent with observations of fluidized bed behaviour: [11] enables us to put these reservations on a quantitative footing.

3.1.3. The intermediate regime

Figure 1 shows numerical evaluations of the voidage function, $g(\varepsilon)$, for a range of values of Re. These were obtained as follows from [11].

First, a value of Re was selected and the unhindered drag coefficient, C_{D0}, corresponding to this value was evaluated from [5]. A value of the void fraction, ε, was then selected and the corresponding value of Re_t evaluated by iteration on the following equations:

$$\frac{\mathrm{Re}}{\mathrm{Re}_t} = \varepsilon^n \qquad [19]$$

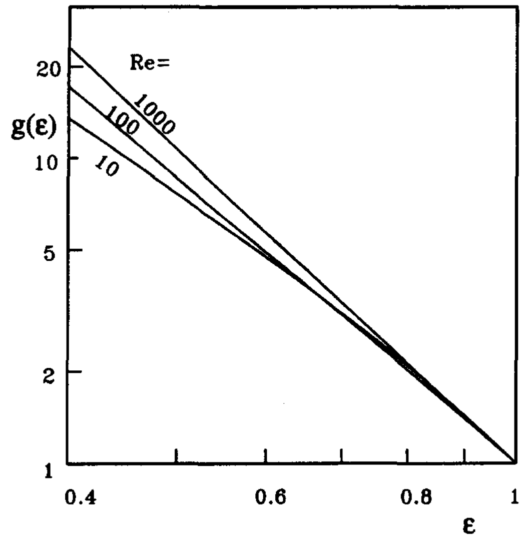

Figure 1. The voidage function for fluidized bed systems at selected values of Re.

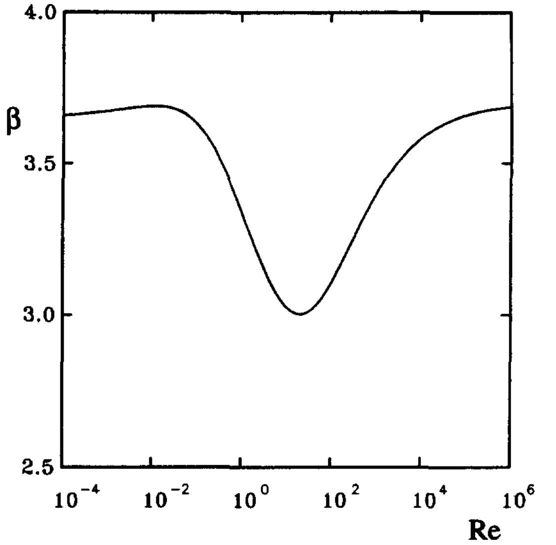

Figure 2. The exponent β for fluidized bed systems as a function of Re.

and

$$\frac{4.7-n}{n-2.35}=0.175\mathrm{Re}_t^{0.75}. \quad [20]$$

Equation [19] is, of course, the Richardson & Zaki (1954) equation and [20] is the convenient continuous form of the original correlation for n proposed by Rowe (1987). The drag coefficient, C_{Dt} in this case, was then obtained from [5] using the evaluated Re_t value and employed in [11] to deliver one point on the $g(\varepsilon)$ vs ε plot. The procedure was then repeated for other values of ε.

It will be seen that the logarithmic plots of figure 1 approximate to straight lines, thereby enabling the voidage function to be expressed as

$$g(\varepsilon)=\varepsilon^{-\beta}, \quad [21]$$

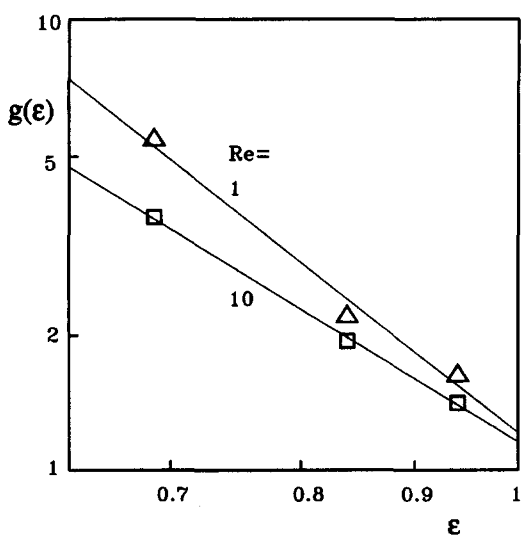

Figure 3. The voidage function evaluated from the data of Happel & Epstein (1954). The solid lines represent the best linear fit of the experimental values.

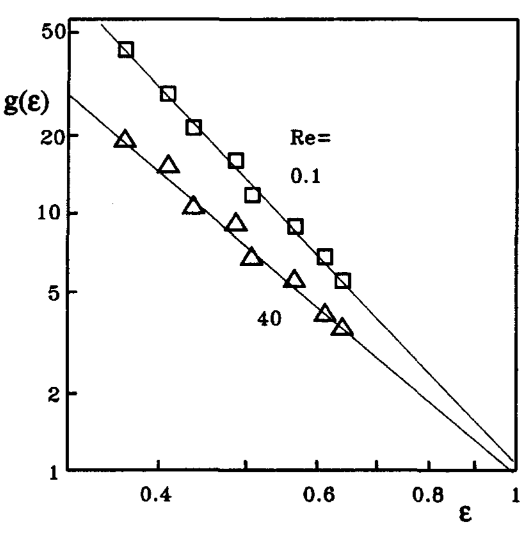

Figure 4. The voidage function evaluated from the data of Rumpf & Gupte (1971). The solid lines represent the best linear fit of the experimental values.

where β depends only on Re. Figure 2 reports evaluations of β over the full working range of Re: it will be seen that β departs from the constant value of about 3.7 found in the low and high Re regimes, passing through a minimum of about 3.1 at an Re of about 30.

3.2. Fixed Beds

The action of the drag force between the particle and fluid phases results in energy dissipation and, as a consequence, a reduction in pressure in the fluid as it passes through a bed, in addition to any change brought about by a change in elevation.

For fixed beds, numerous data for this "piezometric" pressure loss, ΔP, are available and may be used for the evaluation of the voidage function. In order to apply [6] for this purpose, it is first necessary to relate the particle drag force, F_D, to the piezometric pressure loss as follows.

Equation [2], for a fixed bed in which there are particle–particle contact forces, [3], can be written as

$$F_D - V\frac{dp}{dz} = V\rho_p g - F_C. \quad [22]$$

The left-hand side of [22] represents the mutual fluid–particle interaction force which acts on a particle and, in the opposite direction, on the fluid. Thus, the momentum equation for a unit volume of the fluid phase may be written as

$$-\varepsilon\rho g - \left(F_D - V\frac{dp}{dz}\right)\frac{1-\varepsilon}{V} - \frac{dp}{dz} = 0. \quad [23]$$

Rearrangement of [22] and introduction of the piezometric pressure gradient,

$$\frac{dP}{dz} = \frac{dp}{dz} + \rho g, \quad [24]$$

yields the required relationship for a bed of spheres:

$$F_D = \frac{\pi d^3}{6}\left(\frac{\varepsilon}{1-\varepsilon}\right)\frac{\Delta P}{L}, \quad [25]$$

where

$$\Delta P = -L\frac{dP}{dz}. \quad [26]$$

Equation [25] may also be derived from energy balance considerations (Foscolo *et al.* 1983).

Inserting this expression for F_D, together with the drag coefficient expression for F_{D0}, [4], into the definition of the voidage function, [6], yields

$$g(\varepsilon) = \frac{4}{3C_{D0}}\frac{\varepsilon}{1-\varepsilon}\frac{d\Delta P}{\rho u^2 L}. \quad [27]$$

For a given value of Re, C_{D0} may be evaluated, as before, from [5]; empirical data for the dimensionless piezometric pressure loss, $d\Delta P/\rho u^2 L$, as a function of Re and ε then enable $g(\varepsilon)$ to be evaluated from [27].

The vast majority of the empirical data for fixed beds relates to randomly packed spheres for which the void fraction hardly varies from a value of about 0.4; we will see below how, in spite of this obvious limitation, a tentative estimate of how the voidage function varies with the voidage fraction can be made. First, we consider data obtained for fixed beds of spheres in which, by one means or another, the void fraction has been made to vary over a considerable range. These data have been examined previously in a somewhat different context (Gibilaro *et al.* 1985); they include the wind-tunnel results of Wenz & Thodos (1963), obtained using spheres connected together in fixed arrays, which have not been used in the present study as no way is provided to account for the effect of the connecting structure: because of this, extrapolated values of $g(\varepsilon)$, at $\varepsilon = 1$, are about 2.5 instead of the correct value of 1.

3.2.1. The data of Happel & Epstein (1954)

In these experiments fixed, cubic arrangements of spheres were constructed using beads threaded on to slender rods; the resulting voidage fractions covered the range from 0.69 to 0.94. Glycerol solutions were passed through these beds at Re values between 0.4 and 11 and the results reported by means of an empirical equation for the piezometric pressure loss as a function of Re and ε.

Figure 3 shows logarithmic plots of $g(\varepsilon)$ vs ε obtained using [27] for two parametric values of Re: these will be seen to lie on straight lines with an intercept close to 1 at the single particle ($\varepsilon = 1$) limit, yielding β values of 3.9 and 3.1 at Re values of 1 and 10, respectively.

3.2.2. The data of Rumpf & Gupte (1971)

These experiments involved beds of random packed spheres at void fractions of up to 0.64 held together by adhesion at contact points. Numerical values for the dimensionless pressure drop, $d\Delta P/\rho u^2 L$, were provided for both gas and liquid fluxes at Re values between 0.01 and 100.

Figure 4 illustrates examples of $g(\varepsilon)$ relationships obtained from these data: once again the intercept at $\varepsilon = 1$ approximates to 1 in every case, with β values ranging from 3.6, in the low Re regime, to about 3.05.

3.2.3. The Ergun (1952) correlation

The extensive data for flow through random packed spheres, for which $\varepsilon \cong 0.4$, are well-correlated by the Ergun equation:

$$\frac{d\Delta P}{\rho u^2 L} = \frac{1-\varepsilon}{\varepsilon^3}\left[\frac{150(1-\varepsilon)}{\mathrm{Re}} + 1.75\right]. \qquad [28]$$

Applying this relationship to [27] yields

$$g(0.4) = \frac{14.6}{C_{\mathrm{D0}}}\left(1 + \frac{51.4}{\mathrm{Re}}\right). \qquad [29]$$

If we now postulate a relationship of the form $g(\varepsilon) = \varepsilon^{-\beta}$, β may be computed from the extreme values: [29] for $\varepsilon = 0.4$ and $g(1) = 1$.

4. RESULTS AND CONCLUSIONS

Figure 5 shows all the numerical calculations of the voidage function obtained as described above. It is evident that the fixed-bed results (from both the Ergun equation at $\varepsilon = 0.4$, and the

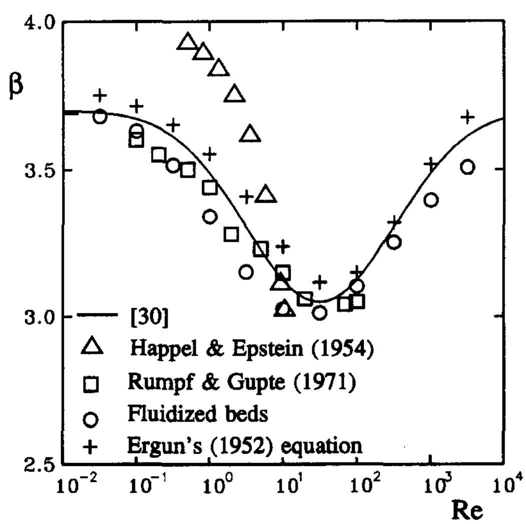

Figure 5. The exponent β evaluated from all the data reported in this work and from [30] as a function of Re.

expanded fixed-bed data) are consistent with those for fluidized beds: a clear minimum is revealed at a Re in the range 20–80. All the results are reasonably fitted by the following empirical expression for the exponent β:

$$\beta = 3.7 - 0.65 \exp\left[-\frac{(1.5 - x)^2}{2} \right], \qquad [30]$$

where $x = \log(\text{Re})$. The effect of this variation in β across the practical range of operation of fluid–particle interaction processes is unlikely to be of major significance but should, nevertheless, be taken into account.

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